Lesson 3

DIRECT AND ALTERNATING CURRENTS

Task. The skills and knowledge taught in this lesson are common to all missile repairer tasks.

Objectives. When you have completed this lesson, you should be able to explain the characteristics of direct and alternating currents; the generation of a sine wave of voltage or current; vector representation of a sine wave; and peak, average, and effective values of alternating current.

Conditions. You will have this subcourse book and work without supervision.

Standard. You must score at least 70 on the end-of-subcourse examination that covers this lesson and lessons 1, 2, and 4 (answer 27 of the 38 questions correctly).

DEFINITION

Direct Current

A direct current (DC) exists when a number of electrons move in one direction only, from a point of low potential to a point of high potential. For example, when the circuit of a flashlight is completed by the switch, electrons continue to move from the negative terminal toward the positive terminals through the filament of the bulb, as long as the switch is on.

Alternating Current

In an alternating-current (AC) circuit, the electrons move through the circuit in one direction for a short period of time, and then move back in the other direction for a like period of time, changing direction at definite intervals. It is this characteristic that gives it its name. The electrons also move toward a point of higher potential, but this point changes from one side of the circuit to the other. In other words, the polarity of the applied alternating voltage changes causing the direction of current flow to change. The two systems are illustrated in figure 3-1.
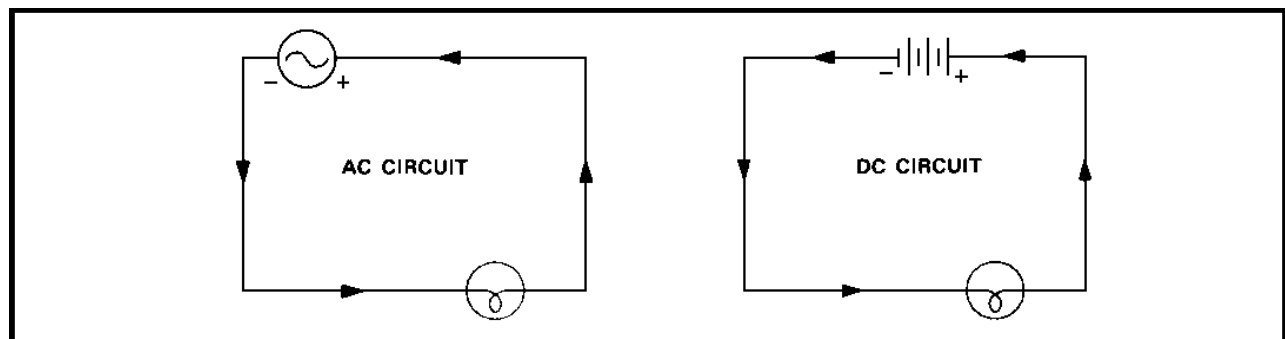

Figure 3-1. AC and DC Circuits.

History

In the beginning of the electrical age, only DC existed. It worked well because energy was consumed relatively close to where it was generated. However, as the demand for electric energy increased, energy had to be transmitted over greater distances. Because power is lost in long-distance lines, DC had to be improved upon.

The solution to the problem came on August 31, 1831, when Michael Faraday wound two separate coils of wire upon an iron ring. The ends of one coil were connected to a battery and switch. The ends of the other coil were joined by a long wire, which passed over a compass needle. When the switch was closed completing the battery circuit, the compass needle moved, then oscillated back and forth, and finally stopped in its original position. When the switch was opened, the needle again moved, in the opposite direction, oscillated, and came to rest. This momentary flow of current was completely unexpected, and Faraday had difficulty explaining it. Later he showed that magnetic and electrical actions were reciprocal because it is necessary for electricity to move before a magnetic effect is produced, and it is necessary for magnetism to move (or change) before an electrical effect is produced. Based on this discovery, AC systems were soon developed that greatly reduced this power loss during transmission. Alternating current systems work so well, they are now used almost exclusively in powerlines.

All forms of communications are based upon variations in the AC transmitting medium that can, for instance, mirror the variation in sound waves your voice produces. In order to communicate by telephone or radio, sound waves must be made to cause electrical variations in the transmitter and receiver. Alternating currents can accommodate these electrical variations. Thus can the intelligence in the sound waves be transmitted electrically.

Direct current is simple, but the generation and characteristics of AC are complex. The rest of this lesson is on AC.

**GENERATION OF CURRENT AND
VOLTAGE BY AN AC SYSTEM**

Principles

An AC generator converts mechanical energy into electrical energy. It does this by the principle of electromagnetic induction. A review of this principal follows.

In the section on magnetism, you learned that a current-carrying conductor produces a magnetic field around itself. You also learned that when a current-carrying conductor is placed in a magnetic field, a force acts on the conductor. This force is perpendicular to the conductor and to the lines of force in the magnetic field. Conversely, if a conductor is moved across the lines of force in a magnetic field, current is caused to flow in the conductor. This current flow also results when the magnetic field is moved with respect to the conductor or when both are moved with respect to each other.

The direction of current flow depends upon the relative motion of the conductor and magnetic field. The amount of current depends upon the strength of the field, speed of relative motion of the conductor and field, and the angle at which the conductor cuts the magnetic lines of force (flux).

Even though current flows through the conductor because the conductor is in motion in a magnetic field, current does not *just* happen to flow. There must be an EMF applied first. Experiments seem to indicate that an EMF is induced into the conductor from the magnetic lines of force when they are broken by the conductor. This induced EMF forces the free electrons within the conductor to move.

The motion is as indicated by the left-hand rule. Recall from lesson 1 that it is used to determine the direction of current in a conductor. This rule is illustrated in figure 3-2, showing lines of force in a magnetic field.
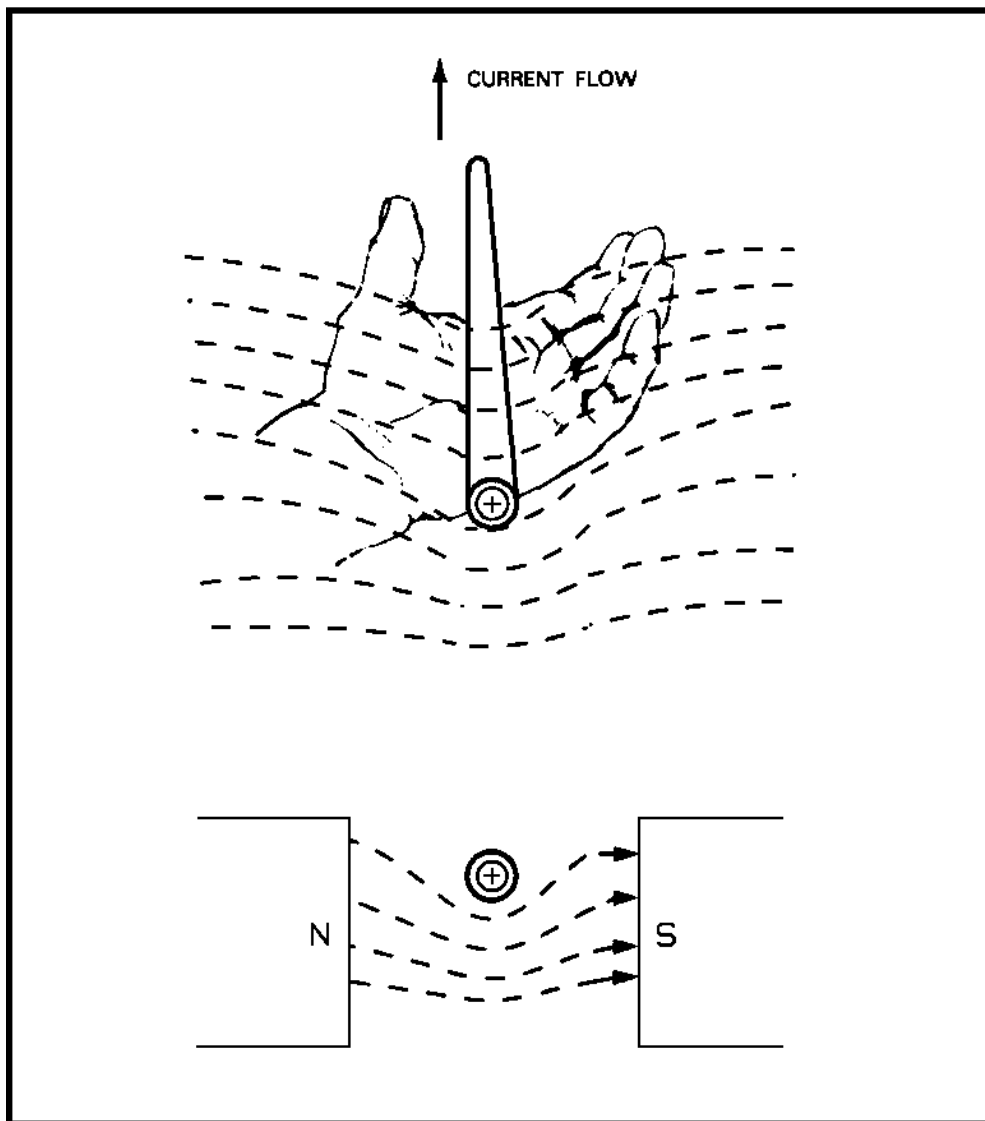

Figure 3-2. Left-Hand Rule Illustrating Current Flow in a Wire, Moving Through a Magnetic Field and Altering Fields of Force.

Generation of AC Voltage

The amount of voltage induced in a coil is dependent upon these factors: strength of the magnetic field, number of turns in the coil, and angle at which the coil cuts the lines of force.

Any movement of the coil in figure 3-3 would induce a voltage in the coil, since it would be cutting the lines of force between the magnetic poles. In figure 3-3A, this single turn coil is at rest in position AB. As the coil rotates in a counterclockwise direction from A toward A' in figure 3-3B, the winding of the coil breaks the lines of force at right angles. This induces a maximum voltage into the coil.
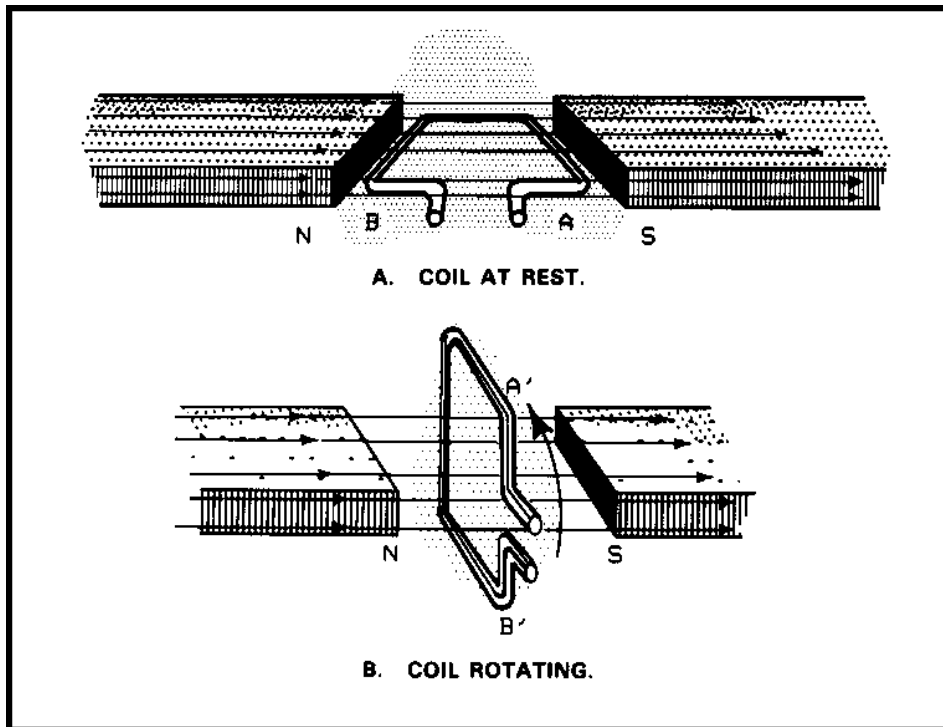

A. COIL AT REST.

B. COIL ROTATING.

Figure 3-3. AC Generator.

The polarity of the induced voltage depends upon the direction in which the field of force is cut. Side A of the coil cuts the lines of force in an upward direction, and side B, at the same instant, cuts the lines of force in a downward direction, producing different polarities. Side A has the negative polarity; side B has a positive polarity. With an external circuit connected to the coil, current would flow from side A through the external circuit to side B. The current inside the coil is from B to A. As the coil rotates from A to A', the angle at which the coil cuts the lines of force is decreased from 90 degrees to 0 degrees. As the angle decreases, the magnitude of the induced voltage decreases until at A'B' the loop is not cutting any lines of force because it is moving parallel to them. At A'B' there is no induced voltage. As the coil continues to rotate, side A moves in a downward direction and side B in an upward direction. The voltage and current flow is of opposite polarity to that which was induced originally. This action explains the first two important concepts of AC: the current is periodically reversing, and the instantaneous values of the current and voltage are constantly changing.

You have seen that a voltage is induced in a conductor if there is relative motion between the conductor and a magnetic field. A machine in which the magnetic field is provided by a permanent magnet is called a magneto. If the magnetic field is provided by an electromagnet, the machine is called a dynamo. The part of the machine which contains the conductors is called the armature. The motion, which causes the induced voltage, can come by rotating the armature, the field, or both.

A generator is a magneto or dynamo which converts mechanical energy to electrical energy. The mechanical energy is supplied by a motor. A motor is a magneto or dynamo that changes electrical energy to mechanical energy or motion.

A simplified generator is shown in figure 3-4. The armature is a rectangular coil rotating between the poles of an electromagnet. (The motor that drives the armature is omitted from the figure.) AB and CD may be considered to be two conductors at the bottom and top of the coil connected to two rings called slip rings or collector rings. These rings are mounted on and rotate with the armature. They make contact with the outer circuit through brushes X and Y. The load on the generator is represented by the resistance R.

As the coil rotates about an axis perpendicular to the magnetic field, in figure 3-4, it cuts lines of force and induces a voltage. In the position shown, no voltage is being induced because the conductors AB and CD are moving parallel to the magnetic field, and no lines of force are being cut. As the coil rotates counterclockwise, AB moves up and CD moves down. By the left-hand
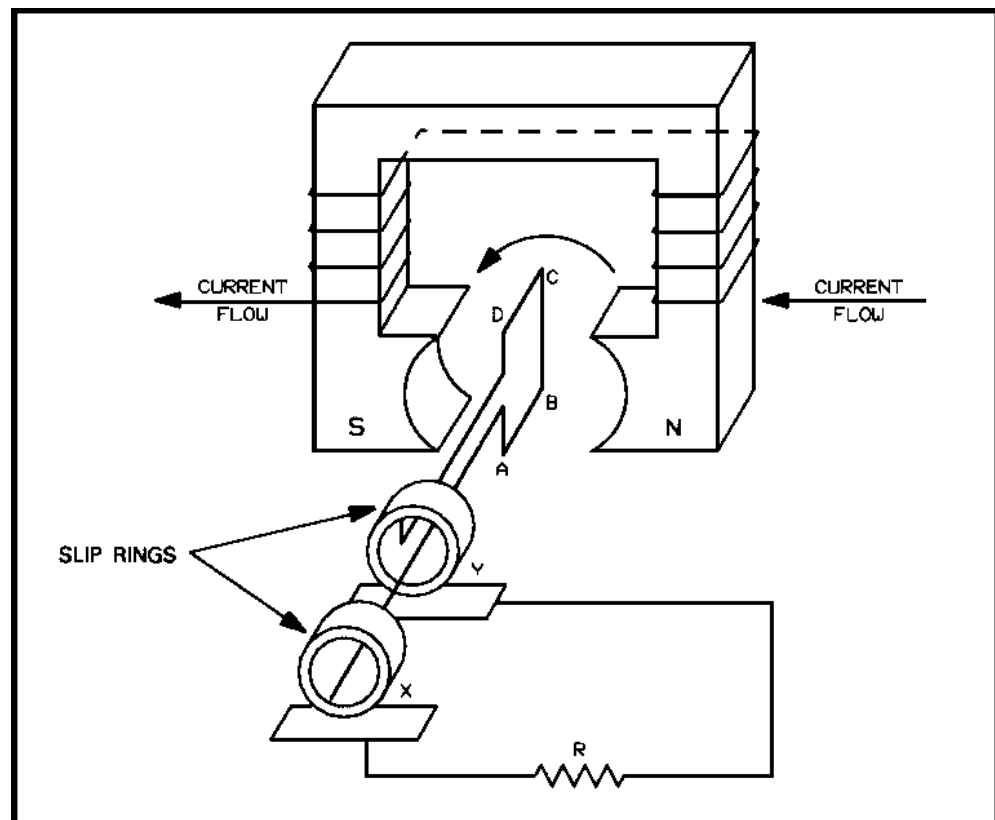

Figure 3-4. Simplified AC Generator.

rule, you can determine that the induced voltage causes electrons to flow from A toward B, in AB, and from C toward D, in CD. Through the outer circuit (R), current flows from Y to X back through the brushes to the conductors. Thus, a complete path is formed for the current with the voltages induced in AB and CD in series. As the coil reaches the point where AB is moving down and CD up, the direction of the electron flow reverses in the conductors and in the outer circuit. Since the currents reverse in conductors AB and CD at the same time, and since the voltage induced in AB is in series with the voltage induced in CD, you can consider them as one conductor for purposes of explanation.

Study an end view of this conductor taken at 45 degree intervals as the coil is rotated. Figure 3-5 shows the conductor rotating clockwise in a magnetic field whose flux lines are from left to right. The entire process is repeated every time the conductor makes a complete revolution.
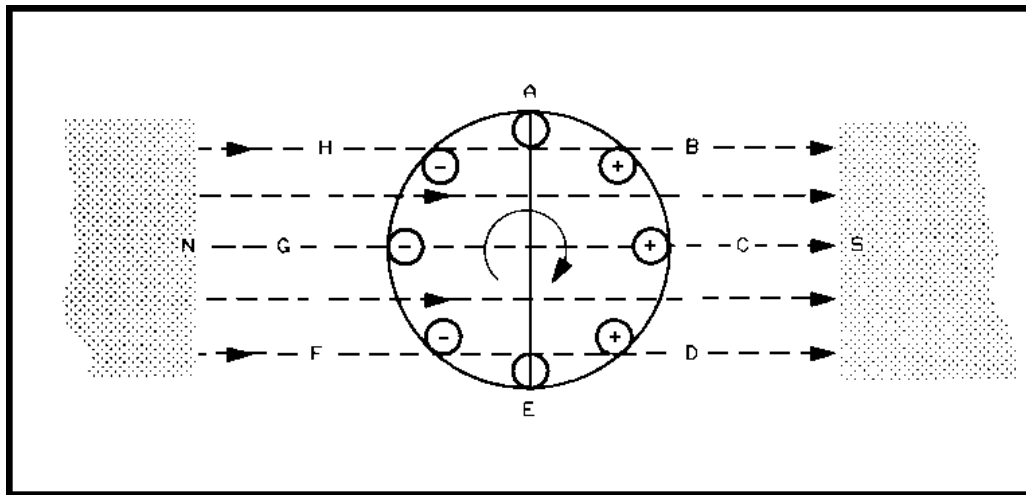

Figure 3-5. End View of a Conductor Rotating Clockwise in a Magnetic Field.

At A. The conductor is traveling parallel to the flux lines; no lines are being cut, and the induced voltage is zero.

From A to B. As the conductor travels from A to B, the number of flux lines being cut per second is increasing; the induced voltage is increasing.

From B to C. The induced voltage continues to increase until C, the maximum. At this point, the conductor is moving perpendicular to the flux lines and cutting a maximum number per second.

From C to D. The number of flux lines being cut per second gradually decreases until point D is reached. At D, the voltage induced is the same as at B.

From D to E. The induced voltage continues to decrease until, at E, it is zero again. At this point, the conductor is again moving parallel to the flux lines.

From E to F. The conductor is now moving upward. The induced voltage is, therefore, in the opposite direction. It increases from zero until, at F, it is equal in magnitude and opposite in polarity to the induced voltage at B or D.

From F to G.   The induced voltage increases until the maximum, at G; this maximum is in a direction opposite to that at C.

From G to H.   The voltage decreases until, at H, it is equal to that at F.

From H to A.   The decrease continues until the conductor returns to its starting position, at A, where the induced voltage is again zero.

If you make a graph of the voltage induced in the conductor at the various points discussed above, you will get a curve (figure 3-6).   Plot the various points in degrees along the horizontal axis and the magnitude of the voltage induced at each point along the vertical axis.   Use the horizontal axis as a reference for zero voltage.   Consider the voltage induced in one direction as positive and that in the opposite direction as negative.
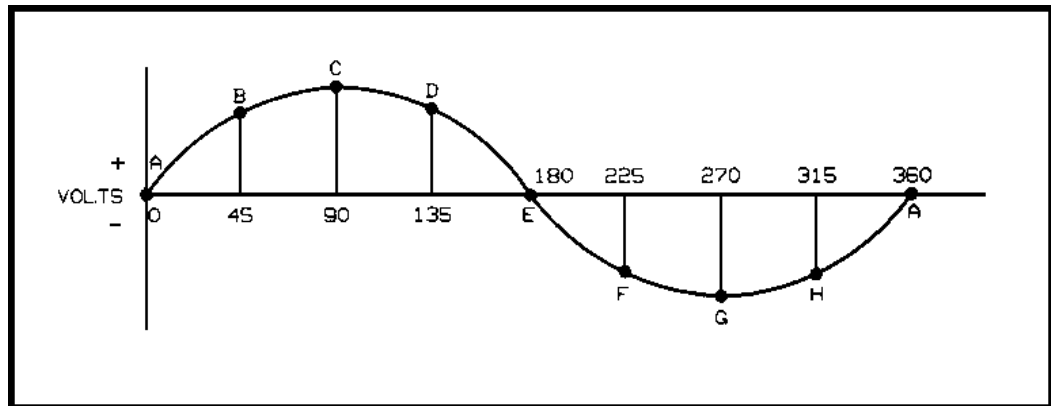

Figure 3-6.   Induced Voltage.

The curve is similar to one developed in mathematics for the variations of the sine function of an angle as the angle is increased.   Since this function is called a sine, the curve is known as a sine wave.   It can also be used to represent the voltage or current produced by a generator of the type discussed. Because this output periodically changes its direction, it is called *alternating current*, and the generator is an AC generator.   Figure 3-7 illustrates a simple AC generator with its armature in various positions.   The sine wave of voltage generated in one revolution of the armature is also shown.   Compare armature positions that differ by 180 degrees, and see that the current has reversed itself.   The usefulness of representing AC generation this way is discussed next.

GRAPHIC REPRESENTATION OF AC

In order to represent this action in graphic form, two factors have to be taken into consideration.   One, the voltage and current are building up from zero to maximum, first in a positive direction, and then in a negative direction.   Thus, voltage or current can be represented by the projections of a rotating vector. Two, it takes time to complete this action.   You will find vectors quite useful in the section of this course that deals with the effects of capacitors and inductors on AC circuits.   They are used to determine AC impedance's and currents, as well as voltages.   Vectors are discussed in detail in the first subcourse of this three-part series.
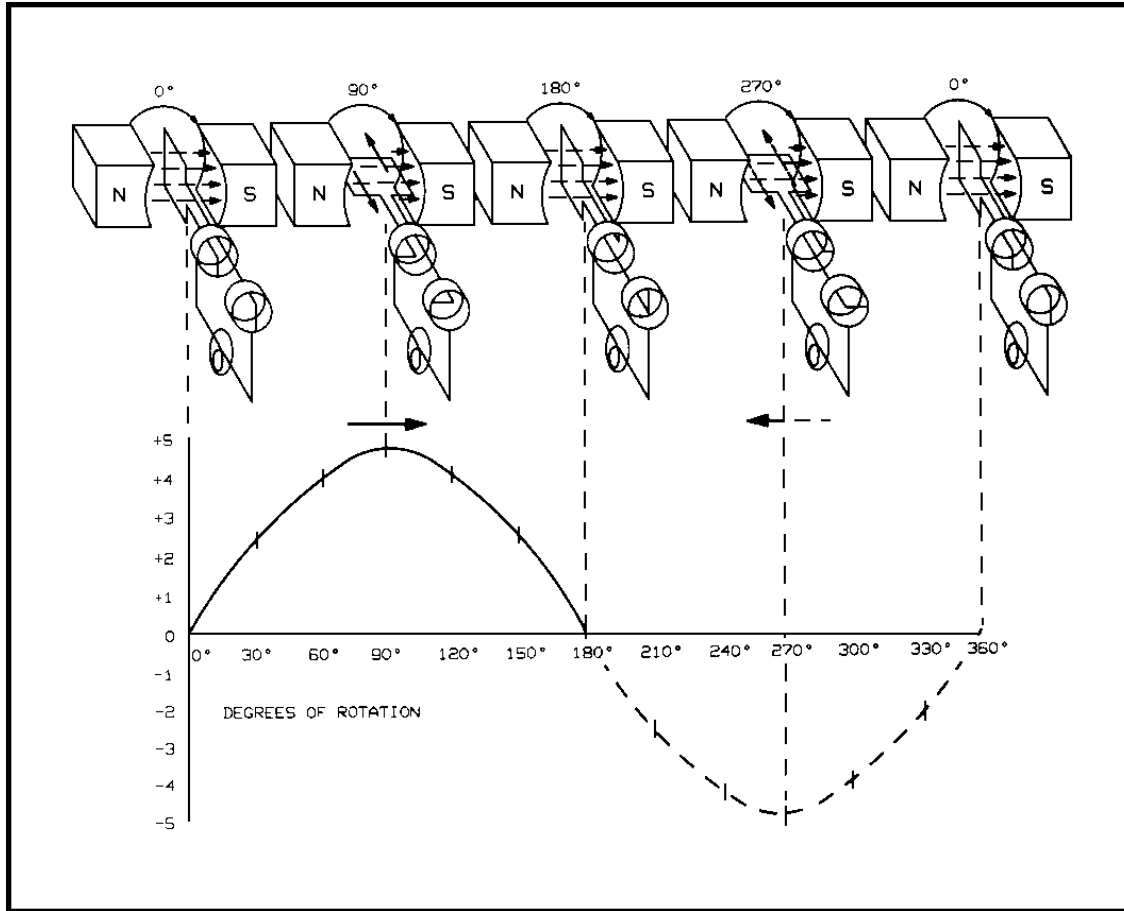
Figure 3-7.   Generation of AC Voltage.

Different positions of the vector are shown in the circles of figure 3-8. Start the vector in the horizontal position, as shown at the top, and rotate it counterclockwise. Each position shown occurs after the vector has rotated 45 degrees from the preceding position. Since it takes time for the vector to rotate, AC voltage can be represented graphically with the horizontal line representing time and the vertical values representing the amounts of voltage, at any time, during the rotation. The length of the vector represents the peak, or maximum, value of the voltage obtained; so, the vertical distance from the end of the vector to the horizontal line will represent the voltage being generated at that instant.

Figure 3-8 shows the development of the graph of the different values of voltages for each position of the vector. In labeling the time values along the horizontal line, the graph starts at the left with zero time. Each point along the time axis indicates the time elapsed since zero time. For example, if it takes 1 sec to make one complete revolution, the graph can be labeled as shown in figure 3-9. Figure 3-10 combines figures 3-8 and 3-9.
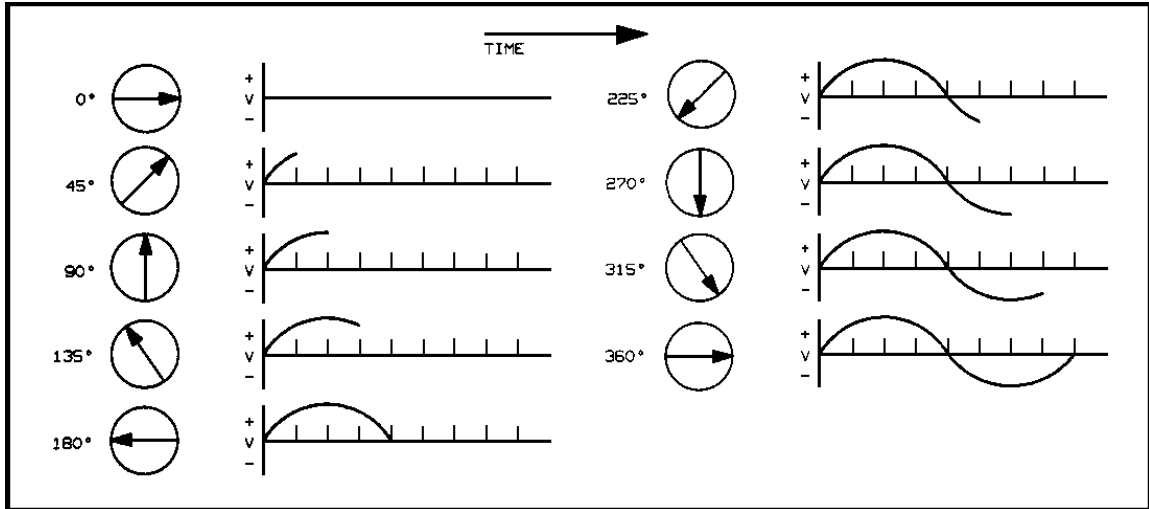
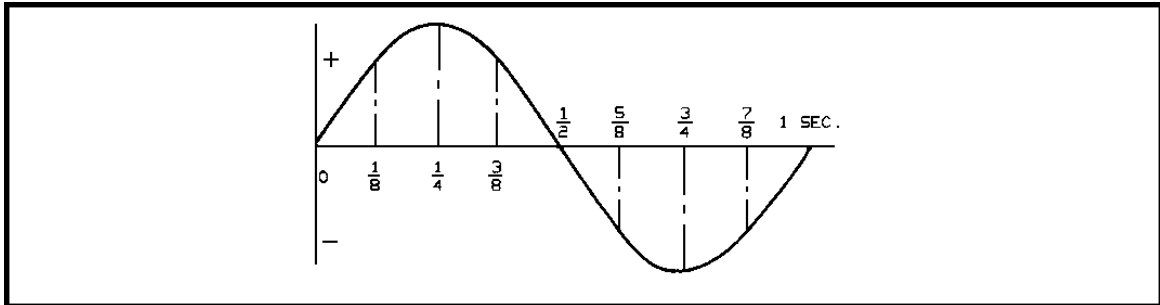*Figure 3-8.   Generation of Sine Wave as Represented by a Rotating Vector.*
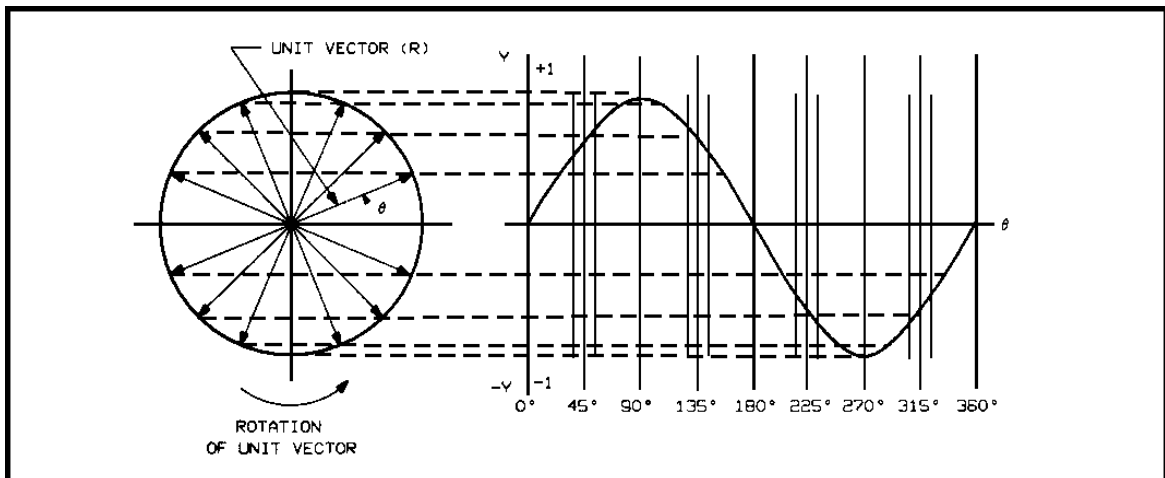

Figure 3-9.   One Cycle per Second.

Figure 3-10.   A Sine Wave in Time.

Actually, an ordinary AC generator rotates many times a second so the time values become very small fractions. Generally, the values along the time axis are in terms of the number of degrees completed since the start of the graph. The values on the time axis may be represented in radians as well as degrees (figure 3-11). The term *radian* is explained in the first subcourse of this three-part series.
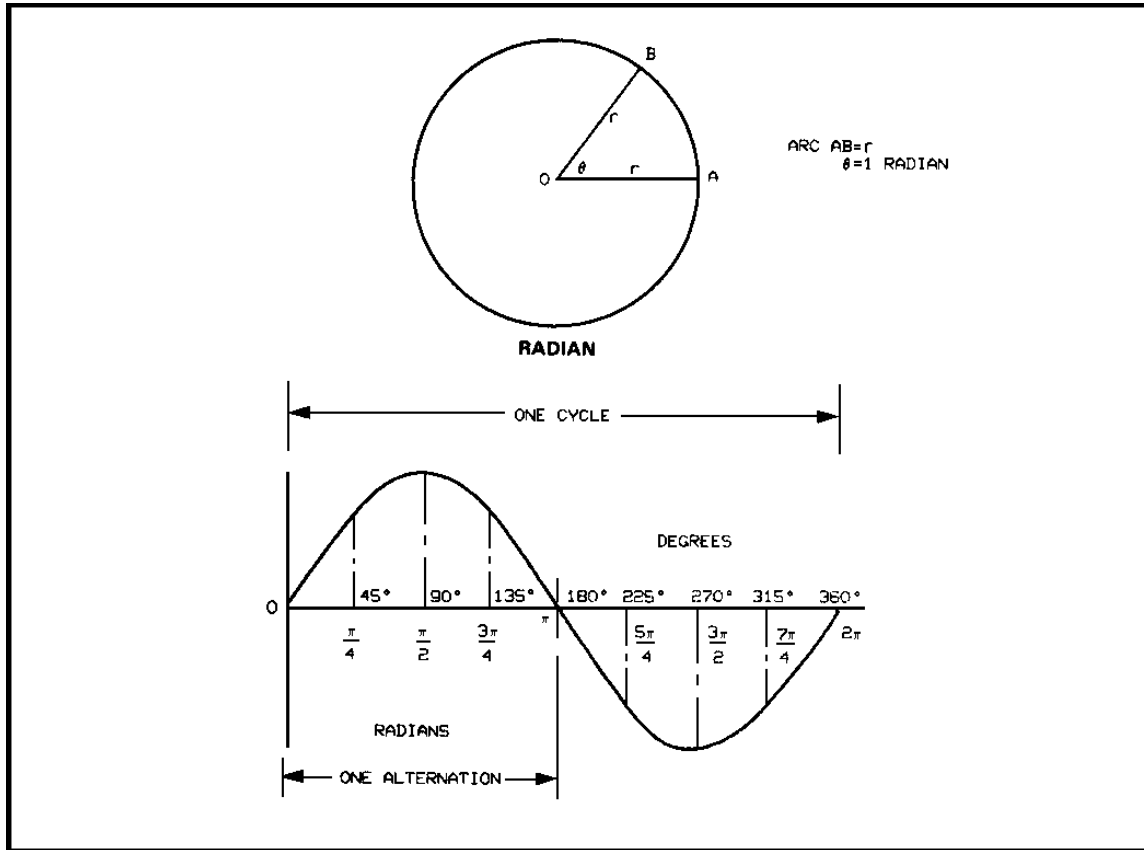

Figure 3-11. Relationship of Radians to Degrees.

The radian, like the degree (°), is a measure of angular distance. Refer to figure 3-11, as you read the following explanation of the radian. This figure shows a circle having its center at O and a radius, r. An arc AB, equal in length to r, is laid off on the circumference of the circle. The ends of the arc AB are connected to the center of the circle with straight lines AO and BO. The angle θ, thus formed by AOB, is equal to one radian. The radian is defined as the angle subtended by the arc of a circle equal in length to the radius of the circle. There are 2π radians in a circle. In mathematics, π is a constant which is equal to 3.14. Understanding the graphing of AC will make the following characteristics and illustrations of AC easier to understand.

FREQUENCY OF AC VOLTAGE AND CURRENT WAVES

The graph of figure 3-11 shows the correlation of radians and degrees for one revolution of a circle. Since this same process is repeated many times, one revolution is referred to as a cycle. Half of a cycle is an alternation. In figure 3-11, all values of the wave between 0 degrees and 180 degrees are above the zero reference line; therefore, that portion of the wave is called the positive alternation. The portion between 180 degrees and 360 degrees is the negative alternation.

Recall the graph in figure 3-9. It shows what happens when it takes one sec to complete a cycle. This is referred to as one cycle per second and is expressed with the term hertz, abbreviated Hz. The AC voltage used in house lighting is 60 Hz. The number of cycles per second of an AC voltage or current is its frequency. The time required to complete one cycle is the period (P = 1/f). The period of a 60 Hz voltage is 1/60 sec.

Every AC voltage generated has some frequency. The frequency studied in this course will range from about 16 to 20 Hz, up into the millions of Hz. The frequencies within this spectrum have many uses (figure 3-12). Most electricity is used for lights, motors, heating devices, etc. When electricity is used for power, its frequency is termed *power frequency*. The most commonly used power frequency is 60 Hz. The lowest frequencies, between 20 Hz and 20,000 Hz, are termed *audio frequencies*, for these are frequencies of vibration that can be heard. For instance, if an AC current, whose frequency is 400 Hz, flows through a voice coil of a loudspeaker, the loudspeaker diaphragm vibrates accordingly producing an audible tone whose frequency is 400 vibrations per second. However, there is a limit to how high a frequency the human ear can hear. This limit for the normal ear is about 15,000 Hz.
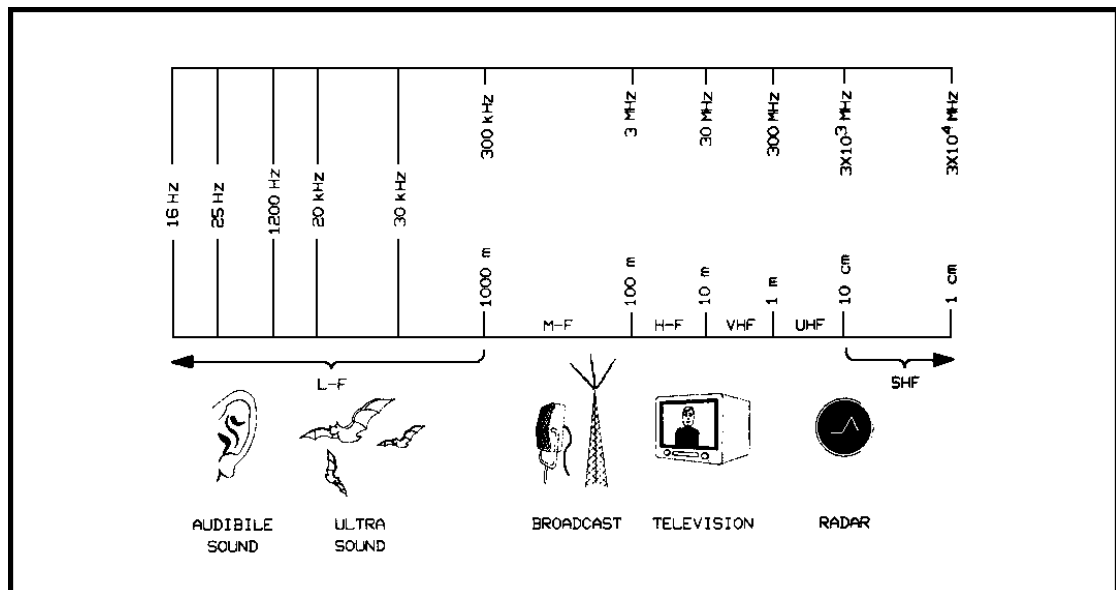

Figure 3-12. Frequency Spectrum.

Frequencies above the audio range are in the thousands and millions of Hz. For convenience, the expression, a thousand Hz, is shortened to a kilohertz (KHz); twenty thousand Hz, then, would be 20 KHz. Megahertz (MHz) represent millions of Hz; therefore, 30 MHz means 30 million Hz. The frequency spectrum shown in figure 3-12 is divided into ranges as follows:

    L-F = low frequencies.
    M-F = medium frequencies.
    H-F = high frequencies.
    VHF = very high frequencies.
    UHF = ultra high frequencies.
    SHF = super high frequencies.

The numbers 1,000m, 100m, 10m, etc., in figure 3-12, refer to the wavelength of the frequencies in meters. For example, one cycle of a voltage which has a frequency of 300 KHz has a length of 1,000m. The length of one cycle of a wave of 3 x 104 MHz frequency is one centimeter (cm) which is equal to 1/100 of a meter. You will notice that the wavelength becomes shorter as the frequency becomes greater. Compare the wavelength of 3 MHz and 30 MHz. On figure 3-12, the 3 MHz frequency has a wavelength of 100 meters, but the 30 MHz frequency has a wavelength of only 10m. This is because of the relationship between frequency and period, which was discussed earlier in this lesson.

ANGULAR MOTION

The linear velocity of a point is the rate at which the point travels along a straight line. The unit of velocity is the distance traveled in a unit of time when the motion of the point is uniform, such as miles per hour (mph), feet per second (fps), and centimeters per second (cps). The same concept is used to measure and define angular velocity.

In figure 3-13, the radius vector (r) is turning about the origin in a counterclockwise direction to generate an angle theta, θ. The angular velocity of such a rotating vector is the rate at which an angle is generated by rotation. When the rotation is uniform, such as in an AC generator, the unit of angular velocity is the angle generated per unit of time. Thus, angular velocity is measured in degrees per second of radians per second.

Radians per second is the more widely used measure of angular velocity. When a rotating vector has completed one revolution, it has gone through an angle of 2π radians. The letter f is used as the abbreviation for the frequency of an AC voltage or current. A rotating vector, then, representing AC, completes f revolutions per second. The angular velocity in radians per second is the number of cycles per second (f) times the number of radians in one cycle (2π). Two π times f, therefore, is the angular velocity in radians per second. The angular velocity in radians per second is denoted by the Greek symbol omega (ω). Expressed in a formula, $\omega = 2\pi f$ radians per second. Find the angular velocity of a vector representing 60 Hz AC. Using 3.14 for π, omega = 2π X f = 2 X 3.14 X 60 = 376.8 radians per second.

The linear distance covered by a point of object moving in a particular direction is its velocity multiplied by the time it has been moving. Similarly, the
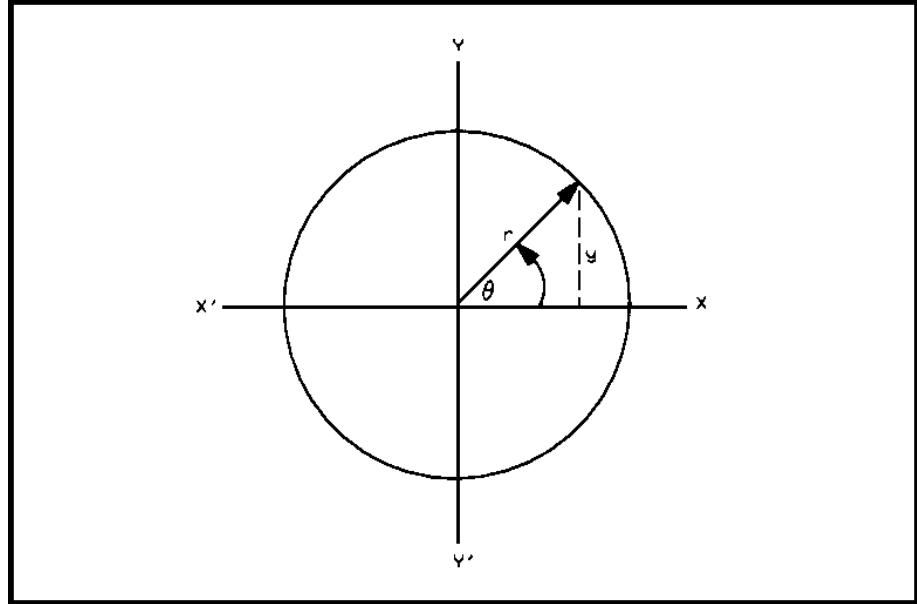
Figure 3-13. Rotating Vector Generating an Angle

angular distance covered by a rotating vector in seconds is its angular velocity (2π x f) times the time (t). This angular distance is the total angle generated. Then, $\theta = \omega$ x t = 2π x f x t radians. Find the angle generated by a 60 Hz AC in 0.01 sec: $\theta = 2\pi$ x f x t = 2 x 3.14 x 60 x .01 = 3.768 radians. Now, find what the angle is in degrees, θ = 360° x f x t degrees. The angle in the problem is θ = 360° x 60 x .01 = 216°.

AMPLITUDE

Determining Amplitude

A graph of the sine values of the different angles, 0 degrees through 360°, using the horizontal axis to represent the angles and the vertical axis to represent the sine values, follows the same pattern as the graph of AC. There is a direct relationship here between the vertical axis and the angle. The vertical projection, y, of the end of a rotating vector, r, in terms of the generated angle, θ is y = r sin θ. (Refer to figures 3-10 and 3-13.) Y is the opposite side of a right triangle where the angle O and hypotenuse (r) are known and the opposite side is computed. The solution of right triangles was covered in the first subcourse of this series, MM0702. This y from any angle is the amplitude of the wave. The amplitude of a sine wave varies as the sine of the angle.

Similar equations can be written about the values of AC current and AC voltage at any instant. The formula for these instantaneous values of voltage is: e = E_{max} x sin θ. E_{max} stands for the maximum voltage induced. To illustrate the above equation, a certain generator induces a peak (maximum) voltage of 100 V. What is the induced voltage at 30° after the time the voltage is zero? The induced voltage at that instant is: e = E_{max} sin θ = 100 x sin 30° = 100 x 0.5 = 50 V.

The formula for the instantaneous value of current is $i = I_{max} \sin \theta$ x I_{max} indicates the maximum value of induced current, and θ is the number of degrees that the armature wire has turned from the position of zero induction.

Since an AC voltage or current keeps repeating the same cycles, a graph of one cycle is usually sufficient to describe completely an AC sine wave of voltage or current. The vertical distance of any point on the voltage curve for the horizontal axis is the instantaneous value of the voltage. For instance, the voltage at $\pi/2$ is maximum, at π it is zero, and at $3\pi/2$ it is maximum in an opposite direction. The graph is convenient for determining voltage at any desired time of the cycle (fig 3-11).

The top of the curve is indicated as the maximum positive value and the bottom of the curve is the maximum negative value. Maximum value often is referred to as peak value. Twice the peak value, or the difference between the maximum positive and the maximum negative, is termed the peak-to-peak value.

Effective AC Voltage

The AC voltage used for the lights in classrooms is 110 volts. An incandescent light bulb lighted with 110-V AC is just as bright as it is when 110-V DC is applied to the bulb. The 110-V AC is just as effective as 110-V DC. The DC is a steady 110 V, but the AC is constantly changing. The peak voltage must be greater than 110 V for the AC to have the same effect as 110-V DC. Actually, the peak voltage of the AC would be approximately 155 V (110 x 1.41).

The 110 volts is called the effective value of AC voltage. The effective voltage is the value used in ordinary usage of an AC voltage. The 110 volts is actually 110 volts effective value. In a 220-V line, the AC has an effective voltage of 220 V. An AC voltmeter is calibrated in effective values. In every case, the peak voltage is more than the effective. The peak voltage can be found by multiplying the effective voltage by the square root of 2, or 1.414. To find the peak voltage of 110 V, multiply 110 x 1.414 which gives 155.5 V peak voltage. $E_{max} = 1.414$ x E_{eff}.

The effective voltage can be determined from the peak by multiplying the peak by 0.707, which is one divided by the square root of 2. For instance, a peak voltage of 100 V would be 100 x 0.707, or 70.7 V effective AC voltage. Figure 3-14 shows the approximate relationship between effective and peak voltage. Note that the effective value is not the same as the average of all the instantaneous values. The effective value of a sine wave of current may be computed to a fair degree of accuracy by taking equally spaced instantaneous values and extracting the square root of their average, or mean, squared values. For this reason, the effective value is often called the root-mean-square (rms) value. $E_{rms} = 0.707$ x E_{max}.

Since the AC or voltage is of sine wave form, the average current or voltage of one cycle is zero, because of the reversal of direction every half-cycle. The term *average value* actually means the average value of one alternation of a sine wave, such as shown in figure 3-14, and may be computed fairly accurately by taking the average of many instantaneous values between two consecutive zero points of the curve. Thus, the average value is equal to the average height of any voltage or current peak (loop). The average value of a
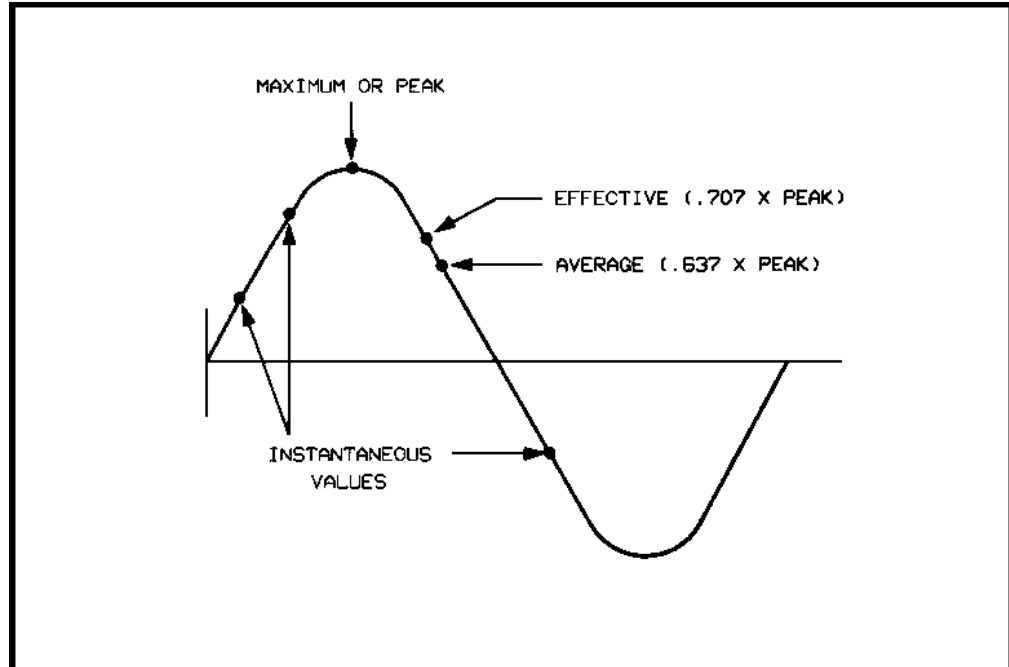
Figure 3-14.   Instantaneous Values.

sine wave of voltage or current is 0.637 times the peak value. E_{avg} = 0.637 E_{max}, and conversely, E_{max} = 1.57 E_{avg}.   The relationship between effective, maximum, average, and peak-to-peak values applies to both AC current and AC voltage.   All AC current meters and AC voltmeters, unless marked to the contrary, read effective values of current or voltage.

PHASE

The phase of a voltage or current wave is very important in the analysis of AC circuits because it is used in comparing two or more AC waves.   The relative instantaneous amplitudes and polarities of a wave represent the phase of the wave.   Two points on a wave are in the same phase if they have the same amplitude and are moving in the same vertical direction, with respect to the zero (reference) point.

The phase of the wave can be expressed in terms of its period.   It is the part of the period which has passed since the wave went through the zero position moving in the positive direction.   Phase may also be expressed in time; that is, as a fraction of a period.   It may also be expressed in angular measure, where one complete cycle represents 360° or 2π radians.   The voltage at points 1 and 13 in figure 3-15 is zero and moving in the positive direction.   These two points are in the same phase.   The phase of each point is zero.   The voltage at point 7 in figure 3-15 is zero and is moving in the negative direction.   Point 7 is 180° out of phase with points 1 and 13.   Point 7 is 180° behind point 1 and 180° ahead of point 13.   The phase of point 7 is 180°.

Point 4 on the curve in figure 3-15 has moved the maximum distance from the reference line in a positive direction.   The phase of this point with respect to point 1 is 90°.   The phase at point 4 lags that at point 1.
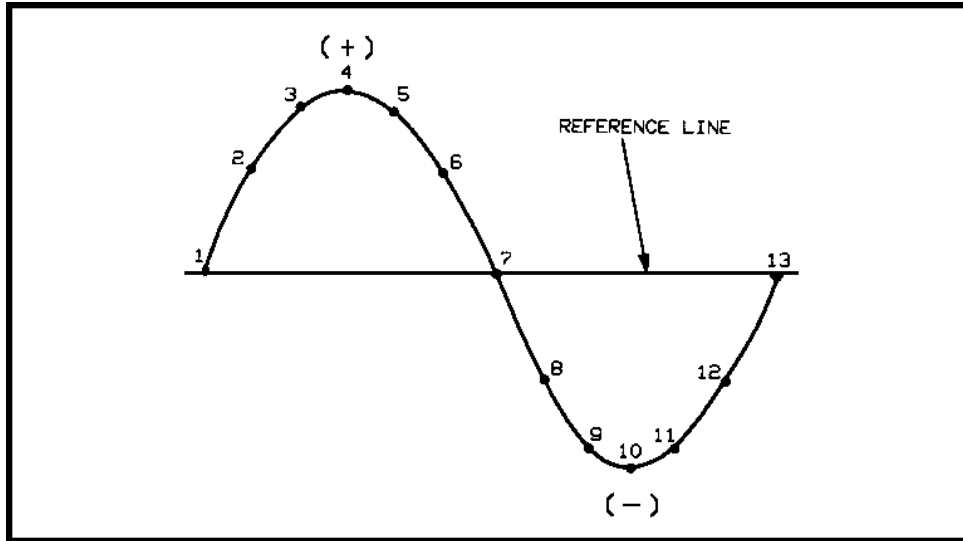
Figure 3-15. Phases.

Point 5 represents the position of the generator armature (30°) after it was at
the maximum positive position (4). This means that point 5 and point 4 have a
phase difference of 30°, or 1/12 of a period or $2\pi/12$ radians. The phase at
point 5 lags that at point 4. Conversely, the phase at point 4 leads that at
point 5.

Points 4 and 10 on the wave in figure 3-15 are 180° out of phase with each
other. Point 4 leads point 10. This phase lead can be expressed as 1/2 the
period or π radians.

Two sine waves generated by two separate generators, which are 180° out of phase
with each other, are shown in figure 3-16. If you compare the phase of each
wave at any given instant, you will see that they are opposite each other in
phase. For example, point 2 is maximum positive on wave A and maximum negative
on wave B. Both waves are zero at points 1, 3, and 5, but notice that the two
waves are moving in opposite directions at each of those points. Point 4 is
maximum negative on A and maximum positive on B. At any given instance, the two
waves are moving in exactly opposite directions.

The outputs of two different AC generators are shown in figure 3-17. These two
waves are 90° out of phase with each other. Point 1 is at zero on wave A and
maximum positive on wave B. At point 2, wave A is maximum positive, but wave B
has fallen to zero. At point 3, wave A has fallen to zero, but wave B has gone
to maximum negative. Wave A is maximum negative at point 4, but B is zero at
this time. Wave A is back to zero at point 5, but B is now at maximum positive
once again. The two waves differ in phase by 90° or 1/4 of the period or $\pi/2$
radians. The same phase difference exists at any given time.

The waves shown in figures 3-16 and 3-17 have the same frequency. That is why
their phase difference remains constant. If waves A and B had different
frequencies, their periods would differ and their phase relationship would
change from one instant to another. For example, assume that wave A, in figure
3-16, has a frequency twice that of B. This means that the period for A is 1/2
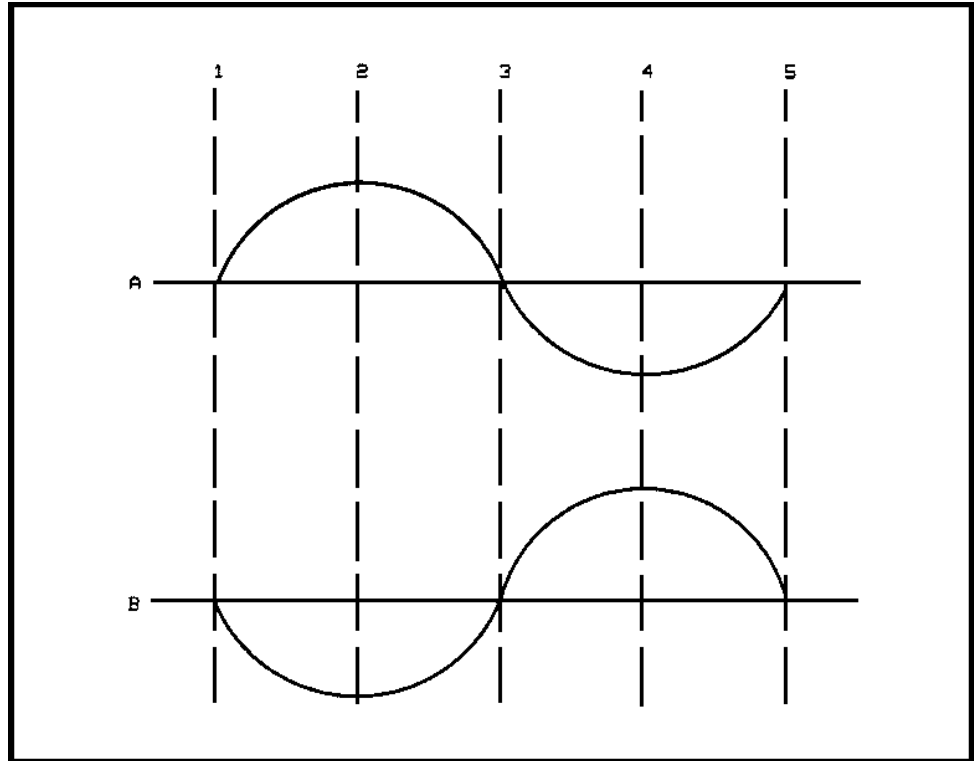the period for B. This also means that, for a given period of time,
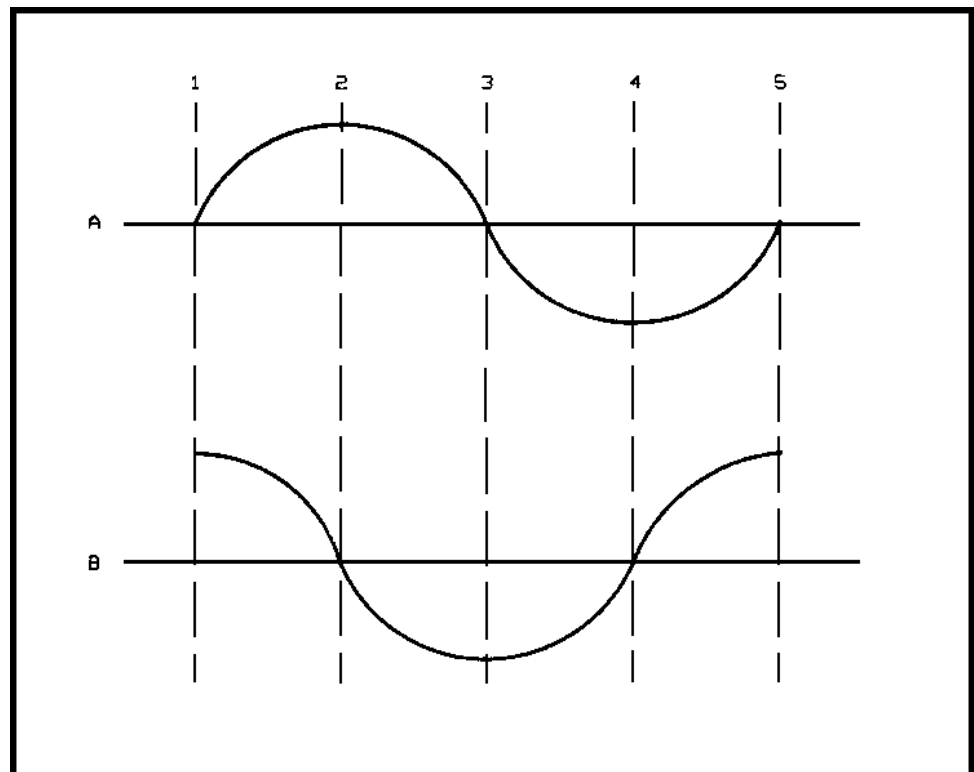
Figure 3-16. 180° Out-of-Phase Sine Waves.

Figure 3-17. 90° Out-of-Phase Sine Waves.

there would be two cycles of wave A for one cycle of wave B. You can see that A would go through its maximum positive value twice while B goes through its maximum positive value only once. The same is true regarding the maximum negative values for the two waves.

COMBINING AC VOLTAGES

Vectors may be used to combine AC voltages of sine waveforms and of the same frequency. The angle between the vectors indicates the time difference between their positive maximum values. The length of the vectors represents either the effective value or the positive maximum value.

The sine wave voltages generated in coils a and b of the simple generators (figure 3-18A) are 90° out of phase because the coils are located 90° apart on the two-pole armatures. The armatures are on a common shaft, and when coil a is cutting squarely across the field, coil b is moving parallel to the field and not cutting through it. Thus, the voltage in coil a is maximum when the voltage in coil b is zero. If the frequency is 60 Hz, the time difference between the positive maximum values of these voltages is

$$\frac{90}{360} \times \frac{1}{60} = 0.00416 \text{ sec.}$$

Vector E_a leads vector E_b by 90° in the parallelogram in figure 3-18B and sine wave a leads sine wave b by 90° in figure 3-18C. The curves are shown on separate axes to identify them with their respective generators; they are also projected on their instantaneous values. If coils a and b are connected in series and the maximum voltage generated in each coil is 10 V, the total voltage is not 20 V because the two maximum values of voltage do not occur at the same time but are separated one-fourth of a cycle. The voltages cannot be added arithmetically, because they are out of phase. These values, however, can be added vectorially.

E_c in figure 3-18B is the vector sum of E_a and E_b and is the diagonal of the parallelogram, the side of which are E_a and E_b. The effective voltage in each coil is 0.707 x 10 or 7.07 V (where 10 V is the maximum), and represents the length of the sides of the parallelogram. The effective voltage of the series combination is 7.07 x $\sqrt{2}$ or 10 V, and represents the length of the diagonal of the parallelogram.

Curve c in figure 3-18C represents the sine wave variations of the total voltage E_c, developed in the series circuit connecting coils a and b. Voltage E_a leads E_b by 90°. Voltage E_c lags E_a by 45°, and leads E_b by 45°.

Counterclockwise rotation of the vectors is considered positive rotation, thus giving the sense of lead or lag. If E_a and E_b (figure 3-18B) are rotated counterclockwise, and their movement observed from fixed position, E_a will pass this position first and 90° later, E_b will pass the position; thus, E_b lags E_a by 90°. If the maximum voltage in each coil is 10 V, the maximum value of the combined voltage will be 10 x $\sqrt{2}$ or 14.14 V. See figure 3-18C. The important point to remember is that two out-of-phase voltages may be resolved into a single resultant value by the use of vectors.

Though generated voltages were used to illustrate the point in this case, the same rules apply to voltage drops as well. Any number of out-of-phase voltages may be combined vectorally, as long as they all have the same frequency; that is, as long as they remain a fixed number of degrees apart, like the two generator loops. Only their magnitudes (length of their vectors) may be different.
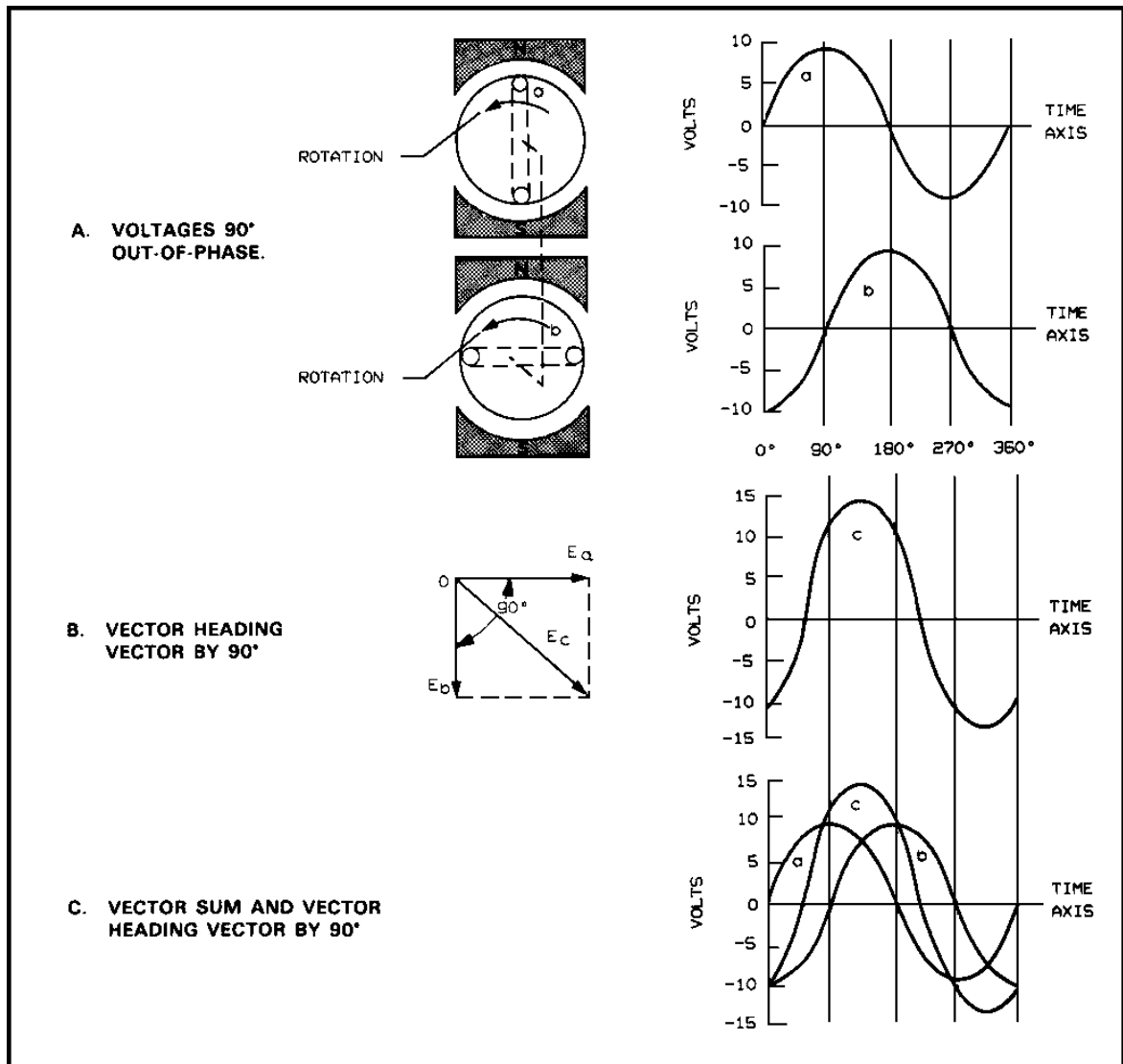

Figure 3-18. Plotting Combined AC Voltages.